## SYNOPSIS

**Title : "A STUDY OF COMPARATIVE STRUCTURE IN FUZZY TOPOLOGICAL SPACES AND ITS APPLICATION"**

### (a) Introduction

We propose to examine how a fuzzy vector topology can be generated from an ordinary vector topology on a given vector space. The focus lies in establishing conditions under which a fuzzy vector topology induced by a family of fuzzy semi-norms becomes fuzzy normable. Rosenfeld introduced the notion of a fuzzy group, which was later extended to fuzzy vector spaces by Katsaras and Liu. The progression from a fuzzy group to a fuzzy vector space parallels the transition from a group to a vector space in the classical setting. we extend the fuzzification of vector addition and scalar multiplication from ordinary vector spaces to their fuzzy counterparts. We also derive the algebraic and geometric concepts of convexity. It is demonstrated that the fuzzy topology induced by a fuzzy seminorm is not always generated topologically. However, a locally convex fuzzy topological vector space are extend to be fuzzy normable through the use of fuzzy seminorms.

### (b) Work plan

We propose to introduce improved concept of pairwise separation axioms and show that the property of being pairwise to-space is a pairwise topological property. We propose to derive the properties of existing separation axioms which has been modified by Sunder Lal with specific properties of invariant under pairwise homomorphism. A common and important class of topological spaces where the distance between points is defined by a metric function, like the Euclidean distance or Hamming distance. The concepts of pairwise continuous and open mapping have been introduced by Kelley and Mukherjee. That area of mathematics which investigates the essential notions of topology such as continuity through the framework of set theory. It provides the foundational definitions and constructions for topological spaces, where sets are endowed with a topology. We propose to generalize the results derived by Neubrunn by using the notion of u-densily, 1-density and density of a collection of subsets. The concept of intuitionistic bitopological space was introduced by coker. C.L Chang introduced fuzzy topology while Mathew study the topological structure involving hesitant fuzzy set.

**(c) Details of the project with chapterisation**

We examne homology and co-homologyin algebrric topology derived by M. Polyak . A homology and cohomology theory for topological quandles are introduced. The relation between these co-homology groups and quandle co-homology groups are derived by Michael Polyak.

We further propose to introduce the concept of pairwise $T_1$ and $T_2$ bitopological spaces in order to achieve a unified framework. We also extend specific notions such as $\delta$-clusters, quasi-uniform pairwise H-compactness, and H-closeness within the context of bitopological spaces. Further more, we investigate the Homotphy theory, Topological invariants pairwise $T_1$ property in bitopological spaces and generalize it to a strictly pairwise $T$-space. The main focus will be on the generalisation of necessary and sufficient condition of compactness and separation axiom to regular as well as normal space. We propose to examine some properties of regular and pair-wise normal space and obtain its completeness. Fleture and patty derived some results on pairwise compact and pair –wise regular bitopolocal space. We propose to derive necessary and sufficient condition for a bitopological space to be regular as well as normal space. The whole proposed research project work is diveded in to four Chapters are as follows.

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| Chapter - 1 | Comparative structure of Topological space and its applications on fix point theorem |
| Chapter - 2 | Convergence properties of metric and normed space |
| Chapter - 3 | Role of Separation axioms in topological space |
| Chapter - 4 | Characterization of compactness in Fuzzy topological Space |

**(d) Hypothesis**

We develop homology and cohomology theories for topological quandles and examine it's relation between quandle co-homology theories. Further, we propose to develop a fixed-point theory of Mathematical analysis to explain some conditions in which maps give solutions. Brouwer's Fixed Point Theorem has broad implications and applications in mathematics. A fixed point of a function is a point that remains unchanged under that function. Brouwer's Fixed Point Theorem states that any continuous function from a closed ball in n dimensional Euclidean space to itself has at least one fixed point derived

by E. Solan. This theorem has significant applications in topology, game theory and economics. it is used to prove the existence of equilibrium points in economic models and to study the behavior of dynamical systems.

## (e) METHODOLOGY

The propose research project is theoretical in nature. We proposed to avail the facilities in the post-graduate department of the university and other Libraries such as Indian statistical institute Kolkata, National Library (Science) Kolkata and Indian institute of Science, Bangalore in course of consulting reference books and journals.

## (f) References


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